A Systematic Approach for Solving Coupled Reluctance Network and Finite Element Models

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Accuracy of reluctance network models can be improved by coupling them to finite element models. However, actually describing the coupling in terms of a system of equations can be difficult and error-prone, especially with large problems. This paper presents a systematic method to generate the system of equations for a coupled reluctance network and a scalar potential finite element problem. The method works with arbitrary network topologies and can be easily extended to nonlinear problems as well. Validity of the approach is demonstrated on a simple synchronous reluctance machine model.

***Index Terms*—Equivalent circuits, finite element analysis, tree graphs.**

I. Introduction

Reluctance networks (RN), or magnetic equivalent circuits, are widely used in the pre-design stage and optimization, where fast computation times are highly valued. The networks are typically solved with the nodal method, where a magnetic scalar potential is solved for each node in the network [1], [2]. While easy to use, the nodal method is known to suffer from poor convergence in nonlinear problems [3]. Furthermore, appropriate placing of field sources into the network for the nodal method can be a difficult task [4], [5].

Faster convergence can be obtained by solving the fluxes rather than potentials [3]. However, this approach is rarely used, probably due to the difficulty involved in forming the necessary flux equations, especially with nonlinear problems. Furthermore, most authors seem to use the mesh-based approach, which only works with planar circuits [3], [5]–[7].

Reluctance networks often fail to predict air-gap and leakage fields accurately. Indeed, recent attention has been paid to coupling reluctance networks to different finite element (FE) formulations [8], [9]. However, most papers have focused on the FE-side of the coupling, only utilizing very simple reluctance networks solved manually. Indeed, it is reasonable to assume that creating a large coupled system by hand will be extremely tedious and error-prone.

This paper presents a systematic, easily programmed approach to automatically form the systems of equations for coupled RN-FE problems. The RN is solved with the fundamental loop method to avoid the problems associated with nodal- and mesh-based approaches. Magnetic scalar potential is used for the FE problem. Behaviour of the coupling is evaluated by simulating a simple synchronous reluctance machine.

II. Methods

The proposed coupling method is described here. A simplified problem in Fig. 1 is used as an example. The part of the network with the nodes 1–6 is a pure reluctance network, while the FE part is presented with the red dashed box.

For an uncoupled RN with n nodes and b edges (reluctances), it is sufficient to define $b-n+1$ linearly independent loops, and associate a virtual flux $\hat{\Phi}_i$ to each. Then, in each of the loops, the sum of magnetomotive force (mmf) drops over the reluctances must equal the total mmf source for the loop. Thus, the fluxes can be solved from

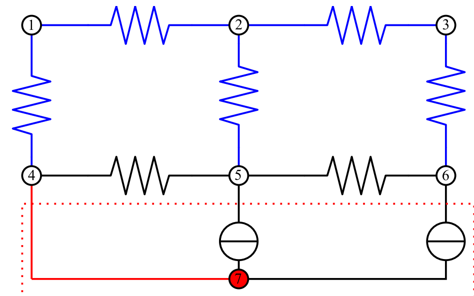

Fig. 1. Example of a coupled reluctance network and FE problem.

$$\mathbf{A}^{\mathrm{T}}\mathbf{D}^{R}\mathbf{A}\hat{\boldsymbol{\Phi}} = \boldsymbol{\mathcal{F}}, \tag{1}$$

where $\mathbf{A}$ is an incidence matrix with the entries

$$\mathbf{A}_{ij} = \begin{cases} 1 & \text{flux } j \text{ traverses edge } i \text{ forwards} \\ -1 & \text{flux } j \text{ traverses edge } i \text{ backwards} \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

and $\mathbf{D}^R$ is a diagonal matrix containing the reluctances. $\hat{\boldsymbol{\Phi}}$ and $\boldsymbol{\mathcal{F}}$ are vectors of the loop fluxes and mmf sources, respectively. The part of $\boldsymbol{\mathcal{F}}$ due to currents can be obtained by calculating the total current enclosed by each loop. The loops and $\mathbf{A}$ can be generated by the well-known fundamental loop method, by forming a spanning tree for the nodes of the network and then connecting all edges *outside* the tree with paths going *through* the tree. In the example, the spanning tree is highlighted in blue, resulting in the loops (4,5,2,1) and (5,6,3,2).

Now, however, it is assumed that some of the nodes (N_{c} in total) of the RN are connected to a scalar potential FE problem

$$\Delta u = f, \quad \mathbf{H} = -\nabla u \quad \Rightarrow \quad \mathbf{S}\mathbf{u} = \mathbf{f}. \tag{3}$$

In the example N_{c} equals 3, with the nodes 4–6 coupled. The potentials $\mathbf{U}^{\mathrm{RN}}$ of the coupled nodes can be used as a Dirichlet boundary condition for the FE problem, so the system is governed by

$$\mathbf{S}_{\mathrm{free}}\mathbf{u}_{\mathrm{free}} + \mathbf{S}_{\mathrm{fixed}}\mathbf{U}^{\mathrm{RN}} = \mathbf{f}_{\mathrm{free}}. \tag{4}$$

From the RN point of view, the FE problem can be presented with a single extra node, N_{c} edges and $N_{\mathrm{c}} - 1$ flux sources. Obviously, the extra node and one of the edges are added to the spanning tree, so a total of $N_{\mathrm{c}} - 1$ additional flux loops (each with a flux source) and the incidence matrix $\mathbf{A}^{\mathrm{FE}}$ are obtained. In Fig. 1, the new spanning tree branch is highlighted in red, resulting in the additional loops (7,5,2,1,4) and (7,6,3,2,1,4).

Due to the extra flux loops, (1) is changed into

$$\mathbf{A}^{\mathrm{T}}\mathbf{D}^{R}\left(\mathbf{A}\hat{\boldsymbol{\Phi}} + \mathbf{A}^{\mathrm{FE}}\hat{\boldsymbol{\Phi}}^{\mathrm{FE}}\right) = \boldsymbol{\mathcal{F}}, \tag{5}$$

The fluxes of the extra loops can be easily obtained by calculating the flux crossing the part of the FE boundary coupled to leaves of the tree, i.e. nodes 5 and 6 in Fig. 1. In general, the expression

$$\hat{\boldsymbol{\Phi}}^{\mathrm{FE}} = \mathbf{M}^{\mathrm{FE}}\mathbf{u} = \mathbf{M}^{\mathrm{FE}}_{\mathrm{free}}\mathbf{u}_{\mathrm{free}} + \mathbf{M}^{\mathrm{FE}}_{\mathrm{fixed}}\mathbf{U}^{\mathrm{RN}} \tag{6}$$

can be substituted back to (5). Additionally, the mmf conservation equation for the extra loops

$$(\mathbf{A}^{\mathrm{FE}})^{\mathrm{T}}\mathbf{D}^{R}\left(\mathbf{A}\hat{\boldsymbol{\Phi}} + \mathbf{A}^{\mathrm{FE}}\hat{\boldsymbol{\Phi}}^{\mathrm{FE}}\right) + \mathbf{C}\mathbf{U}^{\mathrm{RN}} = \boldsymbol{\mathcal{F}}^{\mathrm{FE}} \tag{7}$$

has to be considered. Matrix $\mathbf{C}$ is used to take the potential differences between the coupled nodes into account.

Furthermore, some additional constraints are needed to obtain a solution. Firstly, one of the coupled potentials $\mathbf{U}^{\mathrm{RN}}$ can be fixed to zero to ensure uniqueness. Secondly, if there are N_{d} disjoint reluctance networks connected to the FE domain, $N_{\mathrm{d}} - 1$ flux conservation constraints for the corresponding FE-boundaries are needed.

Finally, if the FE domain is restricted to linear components only, only the diagonal reluctance matrix $\mathbf{D}^{R}$ can be nonlinear. Thus, forming the Jacobian matrix for the Newton's method can be done very easily, by utilizing the chain rule and the equivalent cross-sectional areas of the reluctance elements.

III. Results and Discussion

A simplified four-pole synchronous reluctance machine is simulated both with the coupled method and traditional finite element method. A quarter of the machine and the FE mesh are illustrated in Fig. 2. In the actual simulations, the entire cross-section is modelled, resulting in 2386 nodal potentials to be solved in the FE model.

In the coupled model, only the area highlighted in red is FE-modelled, while the remaining machine is approximated by a simple reluctance network, resulting in only 550 unknowns. Stator teeth and the yoke segments in between are modelled with a single reluctance element each, while four elements are used for the rotor flux barriers.

Fig. 3 illustrates the magnitude of air-gap flux density calculated with the two methods. The results are reasonably accurate, considering the comparative lightness of the coupled model. Further improvements could probably be obtained by refining the reluctance network, especially in the tooth region. Comparison of actual solution times will be included in the full paper.

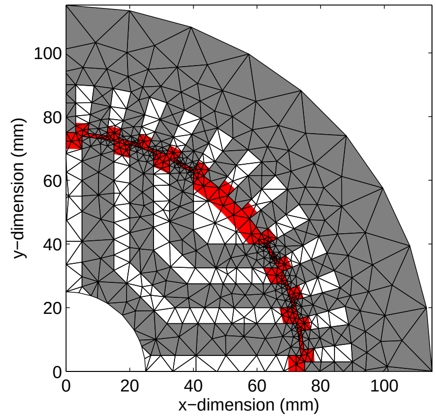

Fig. 2. A quarter of the mesh and the domain used in the coupled FE-model.

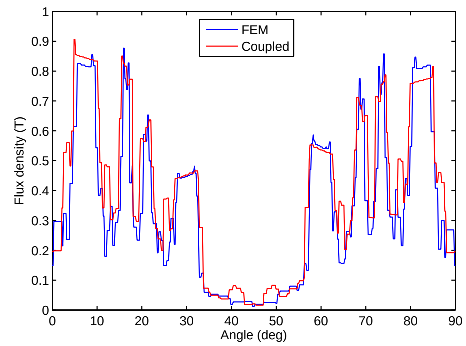

Fig. 3. Amplitude of air-gap flux density over one pole pitch.

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